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EFFECT OF THE ANOMALOUS TEMPERATURE DEPENDENCE OF WATER DENSITY ON SURFACE GRAVITY CURRENT

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This paper reports experimental results on the propagation of a plane water jet at a temperature above the maximum-density temperature (4°C) *along the free surface of initially quiescent water at a temperature of about* 0°C*. For comparison, experiments were performed in which the temperatures of the lower and higher layers were more than* 4°C*, other conditions being equal. The experiments revealed a number of new hydrodynamic effects, including peculiar flow instability and a fine structure of the density field at large times.*

Key words: *maximum water density, hydromagnetic instability, fine structure of the density field.*

The temperature dependence of water density is nonmonotonic. Under normal conditions, the density is maximum at a temperature of about 4°C, and at 0°C, it is 0.07% lower (generally, the temperature of the maximum density $T_{\max}$ depends on the pressure and impurity content). This anomalous property of water, compared to other fluids, is responsible for a number of effects, including hydrodynamic ones. When large masses of water at temperatures above and below $T_{\max}$ come in contact, a layer of higher density surrounded by lower-density water is formed in the contact zone. Under gravity, the water particles in this layer are acted upon by a downward buoyancy force (the difference between the gravity and Archimedes force). This force is small. However, under certain conditions, often occurring in nature, it becomes substantial. In particular, the anomalous temperature dependence of water density plays an important role in heat and mass transfer processes in lake Baikal, where the density stratification is insignificant [1].

The effect of seasonal thermal bar formation in lakes and water basins, in which water masses from the near-shore and deep-water areas of a water body come in contact, has been known since 1880. A review of studies of the thermal bar up to 1992 is contained in [2–4]. Of the later studies mention can be made of [5–9]. Emphasis has been placed primarily on full-scale and theoretical and computational studies. In the case of a thermal bar, the horizontal temperature gradient plays a determining role. The presence of vertical temperature gradients, including those with a layer of maximum density near the bottom, is a typical situation for deep water bodies at any time of year.

The present paper considers some hydrodynamic processes resulting from contact of water masses at temperatures above and below $T_{\max}$ for the flow shown schematically in Fig. 1. A water jet at temperature T_2 was discharged into quiescent water at temperature T_1. In a strongly simplified formulation, this models the propagation of water flowing down a slope in a fresh water reservoir or the propagation of Selenga river water in lake Baikal. A comparison is made with the flow in the absence of anomalous effects with other conditions being equal.

The experiments were conducted in a Plexiglas still-water channel of width $B = 6$ cm and wall thickness 0.6 cm. The channel-bottom shape and longitudinal dimensions are given in Fig. 1. The channel was filled with water at temperature T_1 to a depth of 7.6 cm. Water with temperature T_2 and specific discharge (per unit width of the channel) $q = Q/B$ was let out into this initially quiescent layer. The water was discharged for a limited time, so

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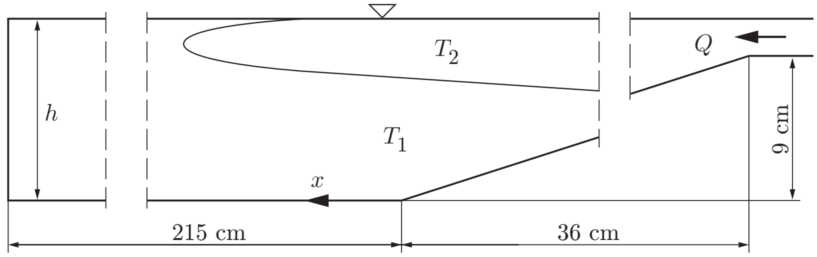

Fig. 1. Diagram of experiment.

TABLE 1

Experiment number	T_1, °C	T_2, °C	q, cm^2/sec	$\Delta\rho$, g/cm^3
1	0.5	11.5	11.1	0.0032
2	11.7	14.2	11.1	0.0031
3	0.2	13.9	23.8	0.0059
4	7.7	13.8	23.8	0.0059
5	11.0	4.0	11.1	0.0038

that during one experiment a total of 10 liters was delivered to the channel and the depth h increased continuously to a value of 14.4 cm. The indoor air temperature was $T = 14.7$°C. In the experiments, the varied parameters were the temperatures T_1 and T_2, the initial density difference $\Delta\rho$, and the specific discharge q. Temperature was measured with an error of ± 0.1°C, and the specific discharge with an error of ± 5%. The values of $\Delta\rho$ are taken from reference books.

Visualization was performed by two methods: using ink and a micron-size aluminum powder. In the first method, information on water motion was obtained over the entire width of the channel, and in the second, it was obtained from a narrower region about 1.5 cm wide separated by a longitudinal light "knife." The observed processes were recorded by a digital photographic camera and a video camera with a frequency of 25 frames/sec. Time t was reckoned with an error of ± 3 sec from the moment the jet entered the quiescent water. The results of five experiment are given below. Their corresponding values of the varied parameters are presented in Table 1. In experiment Nos. 1–4, the jet propagated along the free surface, and in experiment No. 5, over the channel bottom.

The processes developed in time as follows. In the initial interval with a duration of about 8 sec, mixing of warm and cold water occurred over the entire cross section of the channel. In the case of anomalous effects, the mixing was less intense. For the values of q indicated in Table 1, this difference was minor. However, in the experiment with a specific discharge $q = 150$ cm^2/sec, the anomalous effects resulted in a very sharp interface between warm and cold water even in the initial time interval.

Subsequently, in experiment Nos. 1–4, warm water rose to the surface and a surface gravity flow formed. In experiment No. 5, bottom rather than surface gravity flow formed. Transition from T_1 to T_2 occurred in a relatively thin interlayer of thickness δ. The velocity shift between the main layers Δu created conditions for development of Kelvin–Helmholtz instability [10]. In an ideal fluid, this instability occurs at a Richardson number $\mathrm{Ri} = \varepsilon g\delta/\Delta u^2 < 1/4$ [10], where $\varepsilon = \rho_1/\rho_2 - 1$ and g is the acceleration of gravity. In the presence of anomalous effects and with other conditions being equal, the value of δ decreases, so that this instability develops at a smaller velocity shift. In addition, conditions are created for development of Rayleigh–Taylor instability [11, 12].

Figure 2 gives frames obtained in experiment Nos. 1 and 2 at various times before the cessation of water delivery. In experiment No. 1, an aluminum powder was added to the quiescent water and the delivered water was transparent. In experiment No. 2, the quiescent water was transparent and the delivered water was colored with ink. In experiment No. 1 (Fig. 2a and b), the motion was unsteady. In this case, before the cessation of water delivery, both Rayleigh–Taylor and Kelvin–Helmholtz instabilities developed. After the cessation of water delivery, only the Rayleigh–Taylor instability mechanism operated. In the absence of anomalous effects (Fig. 2c and d), the motion was steady-state (except in the initial time interval).

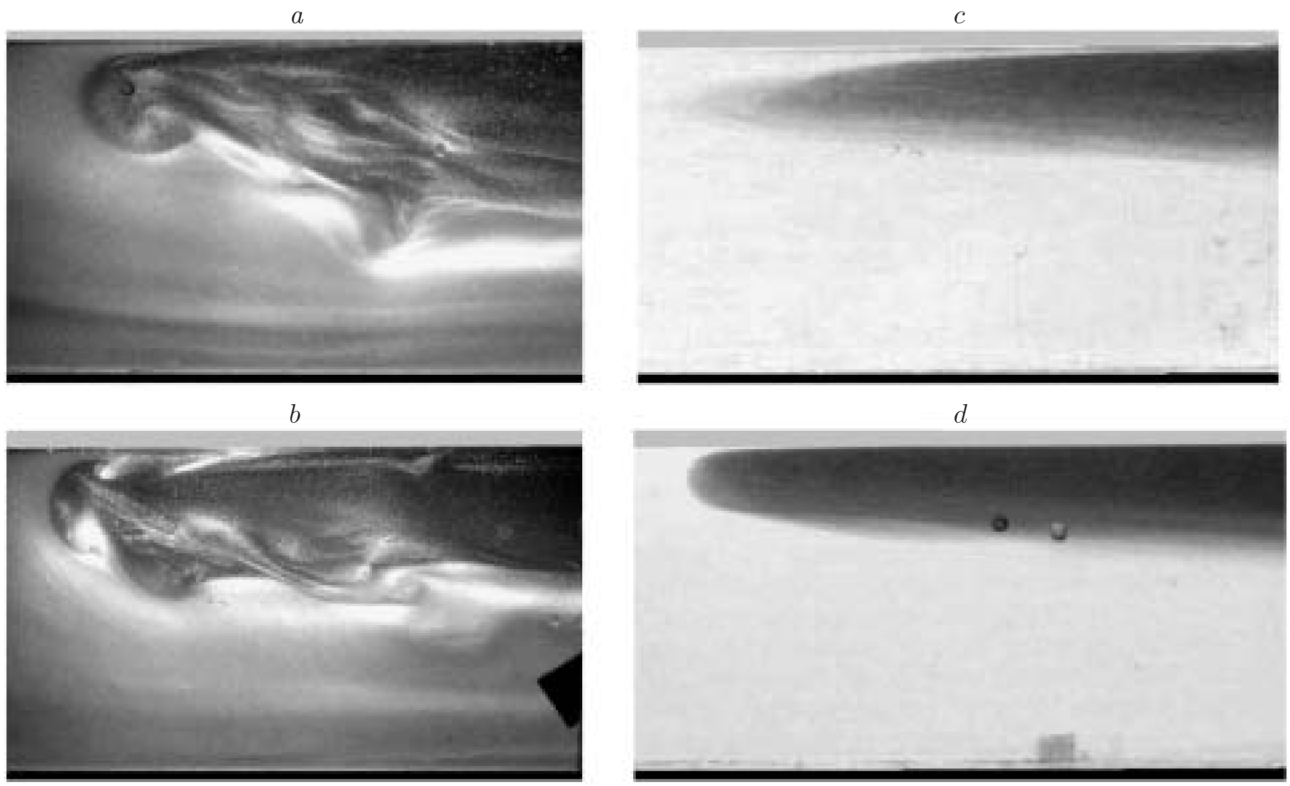

Fig. 2. Head of the surface jet for the lower discharge: (a) experiment No. 1 for $t = 117$ sec; (b) experiment No. 1 for $t = 165$ sec; (c) experiment No. 2 for $t = 89$ sec; (d) experiment No. 2 for $t = 146$ sec.

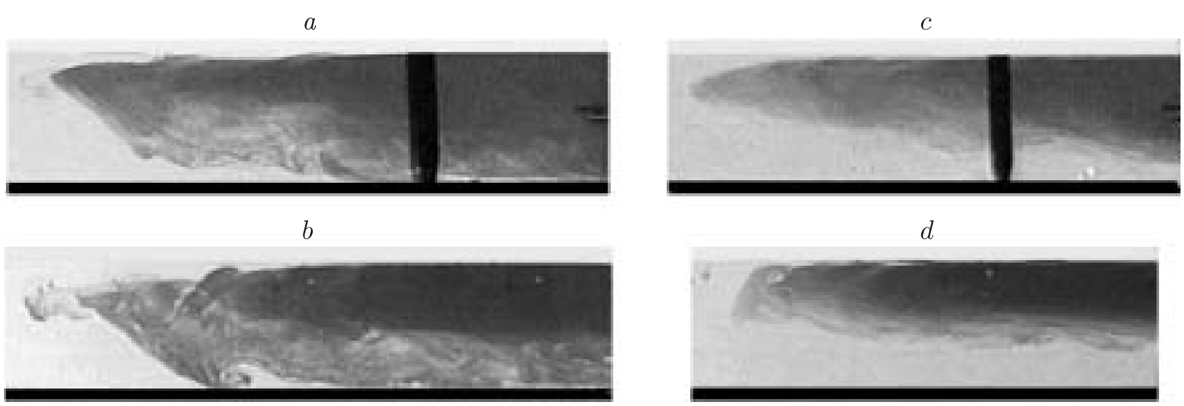

Fig. 3. Head of the surface jet for the higher discharge: (a) experiment No. 3 for $t = 30$ sec; (b) experiment No. 3 for $t = 64$ sec; (c) experiment No. 4 for $t = 29$ sec; (d) experiment No. 4 for $t = 60$ sec.

In the frames, one can see the fine structure of the density field in the surface layer. The term fine structure is taken from [13], where it is introduced to describe a structure with thin horizontal layers of constant density inside a global stable vertical density stratification. The are various causes of fine structure formation. In the present experiments, memory of the previous stages of the process played an important role. In the absence of anomalous effects (Fig. 2c and d), the fine structure is nearly classical [13] and the boundaries of the constant-density layers are continuous. In the presence of anomalous effects (Fig. 2a and b), the fine structure in the jet head is rather peculiar and changes continuously in time.

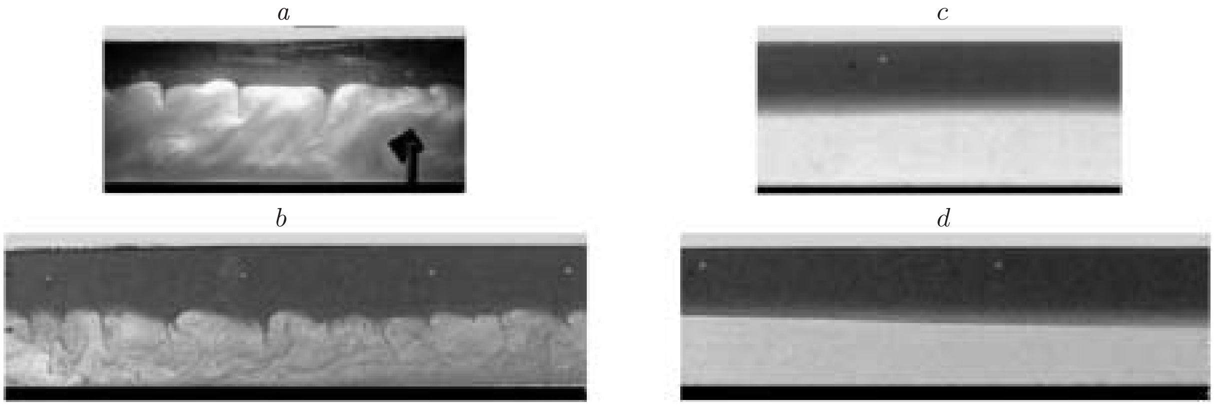

Fig. 4. Flow pattern after the cessation of water delivery: (a) experiment No. 1 for $t = 345$ sec; (b) experiment No. 3 for $t = 164$ sec; (c) experiment No. 2 for $t = 347$ sec; (d) experiment No. 4 for $t = 165$ sec.

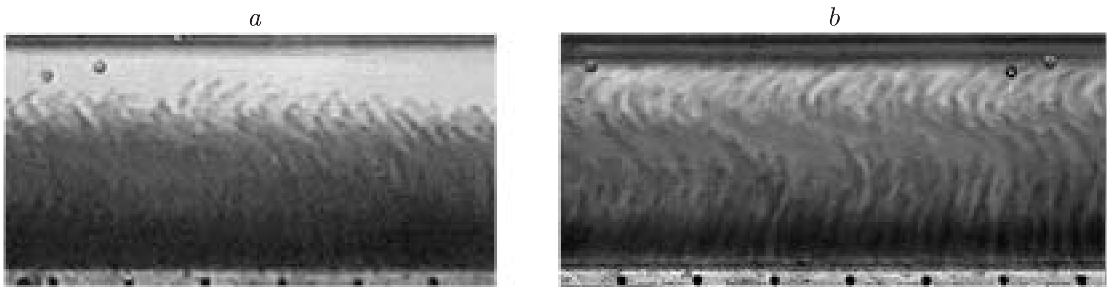

Fig. 5. Fine structure of the density field at large times in experiment No. 5: $t = 2100$ (a) and 3600 sec (b).

Figure 3 gives frames obtained in experiment Nos. 3 and 4. In these experiments, the delivered water was colored with ink. In the presence of anomalous effects, part of the warmer water dipped almost to the channel bottom (Fig. 3a and b). Because of the larger value of Δu, Kelvin–Helmholtz instability also occurred in the absence of anomalous effects (Fig. 3c and d). After the cessation of water delivery, it was rapidly suppressed in both experiments.

The frames in Fig. 4 refer to the time interval after the cessation of water delivery. The frames in Fig. 4 were obtained with visualization using aluminum powder (a) and ink (b–d). In this time interval, the velocity shift is negligible and only the instability due to the anomalous temperature dependence of water density (Fig. 4a and b) is retained. In the absence of anomalous effects (Fig. 4c and d), the stratification is stable and the fine structure has a classical shape.

It should be noted that the instability observed in the frames in Fig. 4a and b differs substantially from Rayleigh–Taylor instability in the case of infinitely deep layers with density inversion. In this case, the excess potential energy of the higher layer is infinite and the flow pattern becomes random with time. In the case of instability due to the anomalous temperature dependence of water density, only a thin layer in the neighborhood of the maximum-density temperature has excess potential energy. Water from this layer dips in the form of quite widely spaced, well-defined jets.

Subsequently, the fine structure of the density field formed (see Fig. 5 for experiment No. 5), which differs significantly from the classical one in that the regularly alternating layers of warm and cold water have a predominantly vertical rather than a horizontal orientation. The discovery of the classical fine structure in oceanology has been widely discussed in the scientific community. It forms in some stages of the evolution of various flows of density-stratified fluids, for example, in the case of convection due to lateral heating of a fluid [14] (see also Fig. 2c and d and Fig. 4c and d).

Additional experiments were performed to verify various hypotheses on the mechanism of formation of the fine structure with vertically oriented layers. In one series experiments, water masses at different temperatures came in contact after removal of the separating vertical partition. In another series, the ice melting process was modeled. In both the basic and additional experiments, a fine structure of the density field similar to that given in Fig. 5 was observed at a certain stage.

The frames given in Fig. 5 are inconsistent with two of the tested hypotheses. One of them stated that the structure in question is due to the presence of ink in the lower layer. However, the ink particles are heavier than water, and in the absence of flows, the fluid colored with ink cannot rise to the surface as concentrated jets, as is the case in the frame in Fig. 5a. Estimates show that the rate of rise of these jets is much higher than the rate of molecular diffusion of ink. The other hypothesis stated that the fine structure observed in the frames in Fig. 5 is due to anomalous effects — when dipping, the transparent water with maximum density displaces the colored fluid upward. However, the picture presented in Fig. 5 was obtained in the absence of anomalous effects (see Table 1). In the presence of anomalous effects, a similar fine structure also forms. Other conditions being equal, this process begins slightly later. One more hypothesis stating that fine structure formation is affected by the heat flow from the laboratory room to the water was not supported in experiments in a larger channel 20 cm wide. From the results of the experiments performed, the following hypothesis seems the most plausible.

The flow occurring after formation of the classical structure with horizontally orientated thin layers plays a key role in the formation of the fine structure with vertically oriented homogeneous layers. The velocity of this flow should be low enough, so that conditions for the well-known blocking effect are created [10]. In this case, the velocity has both a horizontal component and a vertical component of comparable magnitude. Apparently, in the case of a weak global stratification, exactly this factor was responsible for the rise of the heavier fluid and the dipping of the lighter fluid, first in the form of round jets and then in the form of plane vertical jets (Fig. 5). In addition, the formation of so-called fingers due to the different molecular diffusivities of heat and salt was reported [10, 14, 15].

Conditions for the development of the structure in question always exist in deep water bodies such as lake Baikal. The corresponding flows can be caused, in particular, by horizontal temperature nonuniformity. In the present experiments, the weak temperature nonuniformity along the channel (about 1°C per meter of length) due to the prehistory of the motion played a leading role

The rate of vertical transfer of heat and ink in experiment No. 5 (Fig. 5) is much lower than that for turbulent mixing but it is much higher than that for molecular diffusion. Along with other factors, this transfer mechanism may be of significance in the mass-transfer processes in lake Baikal, where the oxygen concentrations in the bottom and surface layers differ only slightly [1]. In describing the processes in lake Baikal, one should also take into account instability due to the anomalous temperature dependence of water density. Because of the very weak density stratification in lake Baikal, even the fact that the maximum-density temperature depends on pressure is of significance.

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